Essential Question: How can you recognize when a pattern in real life is linear or nonlinear?

ACTIVITY: Finding Patterns for Similar Figures

Work with a partner. Copy and complete each table for the sequence of similar rectangles. Graph the data in each table. Decide whether each pattern is linear or nonlinear.

a. Perimeters of similar rectangles

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. Areas of similar rectangles

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Functions

In this lesson, you will
- identify linear and nonlinear functions from tables or graphs.
- compare linear and nonlinear functions.
Section 6.4 Comparing Linear and Nonlinear Functions

2 ACTIVITY: Comparing Linear and Nonlinear Functions

Work with a partner. Each table shows the height \( h \) (in feet) of a falling object at \( t \) seconds.

- Graph the data in each table.
- Decide whether each graph is linear or nonlinear.
- Compare the two falling objects. Which one has an increasing speed?

a. Falling parachute jumper

\[
\begin{array}{c|c|c|c|c|c}
 t & 0 & 1 & 2 & 3 & 4 \\
 \hline
 h & 300 & 285 & 270 & 255 & 240 \\
\end{array}
\]

b. Falling bowling ball

\[
\begin{array}{c|c|c|c|c|c}
 t & 0 & 1 & 2 & 3 & 4 \\
 \hline
 h & 300 & 284 & 236 & 156 & 44 \\
\end{array}
\]

What Is Your Answer?

3. IN YOUR OWN WORDS How can you recognize when a pattern in real life is linear or nonlinear? Describe two real-life patterns: one that is linear and one that is nonlinear. Use patterns that are different from those described in Activities 1 and 2.

Use what you learned about comparing linear and nonlinear functions to complete Exercises 3–6 on page 270.
The graph of a linear function shows a constant rate of change. A nonlinear function does not have a constant rate of change. So, its graph is not a line.

**EXAMPLE 1** Identifying Functions from Tables

Does the table represent a linear or nonlinear function? Explain.

a. 

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>40</td>
</tr>
<tr>
<td>6</td>
<td>32</td>
</tr>
<tr>
<td>9</td>
<td>24</td>
</tr>
<tr>
<td>12</td>
<td>16</td>
</tr>
</tbody>
</table>

As $x$ increases by 3, $y$ decreases by 8. The rate of change is constant. So, the function is linear.

b. 

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td>5</td>
<td>33</td>
</tr>
<tr>
<td>7</td>
<td>88</td>
</tr>
</tbody>
</table>

As $x$ increases by 2, $y$ increases by different amounts. The rate of change is not constant. So, the function is nonlinear.

**EXAMPLE 2** Identifying Functions from Graphs

Does the graph represent a linear or nonlinear function? Explain.

a. 

The graph is not a line. So, the function is nonlinear.

b. 

The graph is a line. So, the function is linear.

**On Your Own**

Does the table or graph represent a linear or nonlinear function? Explain.

1. 

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>25</td>
</tr>
<tr>
<td>7</td>
<td>20</td>
</tr>
<tr>
<td>14</td>
<td>15</td>
</tr>
<tr>
<td>21</td>
<td>10</td>
</tr>
</tbody>
</table>

2. 

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>−4</td>
</tr>
</tbody>
</table>

3. 

The graph is nonlinear.
EXAMPLE 3 Identifying a Nonlinear Function

Which equation represents a **nonlinear** function?

- A) \( y = 4.7 \)
- B) \( y = \pi x \)
- C) \( y = \frac{4}{x} \)
- D) \( y = 4(x - 1) \)

You can rewrite the equations \( y = 4.7 \), \( y = \pi x \), and \( y = 4(x - 1) \) in slope-intercept form. So, they are linear functions.

You cannot rewrite the equation \( y = \frac{4}{x} \) in slope-intercept form.

So, it is a nonlinear function.

\[ \text{The correct answer is (C).} \]

EXAMPLE 4 Real-Life Application

Account A earns simple interest. Account B earns compound interest. The table shows the balances for 5 years. Graph the data and compare the graphs.

<table>
<thead>
<tr>
<th>Year, ( t )</th>
<th>Account A Balance</th>
<th>Account B Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$100</td>
<td>$100</td>
</tr>
<tr>
<td>1</td>
<td>$110</td>
<td>$110</td>
</tr>
<tr>
<td>2</td>
<td>$120</td>
<td>$121</td>
</tr>
<tr>
<td>3</td>
<td>$130</td>
<td>$133.10</td>
</tr>
<tr>
<td>4</td>
<td>$140</td>
<td>$146.41</td>
</tr>
<tr>
<td>5</td>
<td>$150</td>
<td>$161.05</td>
</tr>
</tbody>
</table>

Both graphs show that the balances are positive and increasing.

The balance of Account A has a constant rate of change of $10. So, the function representing the balance of Account A is linear.

The balance of Account B increases by different amounts each year. Because the rate of change is not constant, the function representing the balance of Account B is nonlinear.

On Your Own

Does the equation represent a **linear** or **nonlinear** function? Explain.

4. \( y = x + 5 \)  
5. \( y = \frac{4x}{3} \)  
6. \( y = 1 - x^2 \)
6.4 Exercises

Vocabulary and Concept Check

1. VOCABULARY Describe how linear functions and nonlinear functions are different.

2. WHICH ONE DOESN'T BELONG? Which equation does not belong with the other three? Explain your reasoning.

   \[
   \begin{align*}
   5y &= 2x \\
   y &= \frac{2}{5}x \\
   10y &= 4x \\
   5xy &= 2
   \end{align*}
   \]

Practice and Problem Solving

Graph the data in the table. Decide whether the graph is linear or nonlinear.

3.  

   \[
   \begin{array}{c|c|c|c|c}
   x & 0 & 1 & 2 & 3 \\
   \hline
   y & 4 & 8 & 12 & 16 \\
   \end{array}
   \]

4.  

   \[
   \begin{array}{c|c|c|c|c}
   x & 1 & 2 & 3 & 4 \\
   \hline
   y & 1 & 2 & 6 & 24 \\
   \end{array}
   \]

5.  

   \[
   \begin{array}{c|c|c|c|c}
   x & 6 & 5 & 4 & 3 \\
   \hline
   y & 21 & 15 & 10 & 6 \\
   \end{array}
   \]

6.  

   \[
   \begin{array}{c|c|c|c|c}
   x & -1 & 0 & 1 & 2 \\
   \hline
   y & -7 & -3 & 1 & 5 \\
   \end{array}
   \]

Does the table or graph represent a linear or nonlinear function? Explain.

7.  

   \[
   \begin{array}{c|c|c|c|c}
   x & -4 & -3 & -2 & -1 \\
   \hline
   y & 4 & 3 & 2 & 1 \\
   \end{array}
   \]

8.  

   \[
   \begin{array}{c|c|c|c|c}
   x & -5 & -4 & -3 & -2 \\
   \hline
   y & 4 & 3 & 2 & 1 \\
   \end{array}
   \]

9.  

   \[
   \begin{array}{c|c|c|c|c}
   x & 5 & 11 & 17 & 23 \\
   \hline
   y & 7 & 11 & 15 & 19 \\
   \end{array}
   \]

10.  

    \[
    \begin{array}{c|c|c|c|c}
    x & -3 & -1 & 1 & 3 \\
    \hline
    y & 9 & 1 & 1 & 9 \\
    \end{array}
    \]

11. VOLUME The table shows the volume \( V \) (in cubic feet) of a cube with an edge length of \( x \) feet. Does the table represent a linear or nonlinear function? Explain.

    \[
    \begin{array}{c|c|c|c|c|c|c|c|c}
    \hline
    \text{Edge Length, } x & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
    \hline
    \text{Volume, } V & 1 & 8 & 27 & 64 & 125 & 216 & 343 & 512 \\
    \end{array}
    \]
Does the equation represent a linear or nonlinear function? Explain.

12. $2x + 3y = 7$
13. $y + x = 4x + 5$
14. $y = \frac{8}{x^2}$

15. **LIGHT** The frequency $y$ (in terahertz) of a light wave is a function of its wavelength $x$ (in nanometers). Does the table represent a linear or nonlinear function? Explain.

<table>
<thead>
<tr>
<th>Color</th>
<th>Red</th>
<th>Yellow</th>
<th>Green</th>
<th>Blue</th>
<th>Violet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelength, $x$</td>
<td>660</td>
<td>595</td>
<td>530</td>
<td>465</td>
<td>400</td>
</tr>
<tr>
<td>Frequency, $y$</td>
<td>454</td>
<td>504</td>
<td>566</td>
<td>645</td>
<td>749</td>
</tr>
</tbody>
</table>

16. **MODELING** The table shows the cost $y$ (in dollars) of $x$ pounds of sunflower seeds.

<table>
<thead>
<tr>
<th>Pounds, $x$</th>
<th>Cost, $y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2.80</td>
</tr>
<tr>
<td>3</td>
<td>?</td>
</tr>
<tr>
<td>4</td>
<td>5.60</td>
</tr>
</tbody>
</table>

   a. What is the missing $y$-value that makes the table represent a linear function?
   b. Write a linear function that represents the cost $y$ of $x$ pounds of seeds. Interpret the slope.
   c. Does the function have a maximum value? Explain your reasoning.

17. **TREES** Tree A is 5 feet tall and grows at a rate of 1.5 feet per year. The table shows the height $h$ (in feet) of Tree B after $x$ years.

   a. Does the table represent a linear or nonlinear function? Explain.
   b. Which tree is taller after 10 years? Explain.

<table>
<thead>
<tr>
<th>Years, $x$</th>
<th>Height, $h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>17</td>
</tr>
<tr>
<td>9</td>
<td>23</td>
</tr>
</tbody>
</table>

18. **Number Sense** The ordered pairs represent a function.

   $(0, -1), (1, 0), (2, 3), (3, 8), \text{ and } (4, 15)$

   a. Graph the ordered pairs and describe the pattern. Is the function linear or nonlinear?
   b. Write an equation that represents the function.

**Fair Game Review** What you learned in previous grades & lessons

The vertices of a figure are given. Draw the figure and its image after a dilation with the given scale factor $k$. Identify the type of dilation. **(Section 2.7)**

19. $A(-3, 1), B(-1, 3), C(-1, 1); k = 3$
20. $J(-8, -4), K(2, -4), L(6, -10), M(-8, -10); k = \frac{1}{4}$

21. **MULTIPLE CHOICE** What is the value of $x$? **(Section 3.3)**

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>35</td>
<td>55</td>
<td>125</td>
</tr>
</tbody>
</table>