7.2 Finding Cube Roots

Essential Question How is the cube root of a number different from the square root of a number?

When you multiply a number by itself twice, you cube the number.

Symbol for cubing is the exponent 3.

$4^3 = 4 \cdot 4 \cdot 4$

$= 64$ 4 cubed is 64.

To "undo" this, take the *cube root* of the number.

Symbol for cube root is $\sqrt[3]{\ }$.

$\sqrt[3]{64} = \sqrt[3]{4^3} = 4$ The cube root of 64 is 4.

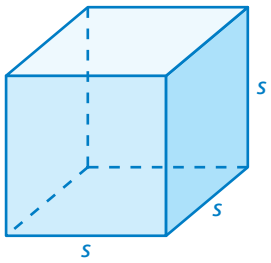
1 ACTIVITY: Finding Cube Roots

Work with a partner. Use a cube root symbol to write the edge length of the cube. Then find the cube root. Check your answer by multiplying.

a. Sample:

$s = \sqrt[3]{343} = \sqrt[3]{7^3} = 7$ inches

Volume = 343 in.^{3}



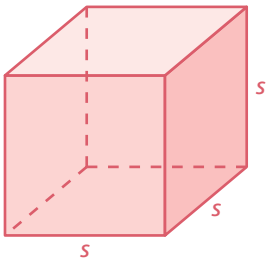
Check

$7 \cdot 7 \cdot 7 = 49 \cdot 7$

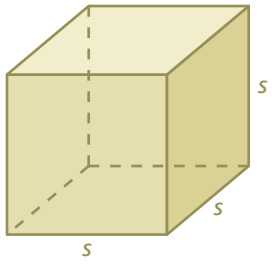
$= 343$ ✓

∴ The edge length of the cube is 7 inches.

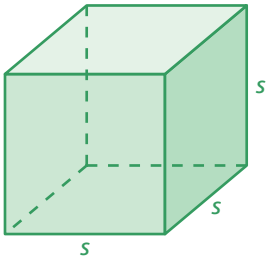
b. Volume = 27 ft^3



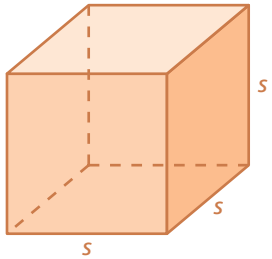
c. Volume = 125 m^3



d. Volume = 0.001 cm^3



e. Volume = $\frac{1}{8}$ yd^3



Cube Roots

In this lesson, you will

- find cube roots of perfect cubes.
- evaluate expressions involving cube roots.
- use cube roots to solve equations.

2 ACTIVITY: Using Prime Factorizations to Find Cube Roots

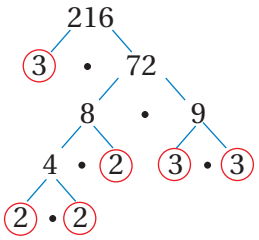
Math Practice

View as Components

When writing the prime factorizations in Activity 2, how many times do you expect to see each factor? Why?

Work with a partner. Write the prime factorization of each number. Then use the prime factorization to find the cube root of the number.

a. 216



$216 = 3 \cdot 2 \cdot 3 \cdot 3 \cdot 2 \cdot 2$ Prime factorization

$= (3 \cdot \square) \cdot (3 \cdot \square) \cdot (3 \cdot \square)$ Commutative Property of Multiplication

$= \square \cdot \square \cdot \square$ Simplify.

∴ The cube root of 216 is $\square$.

b. 1000

c. 3375

d. **STRUCTURE** Does this procedure work for every number? Explain why or why not.

What Is Your Answer?

3. Complete each statement using *positive* or *negative*.

a. A positive number times a positive number is a _____ number.

b. A negative number times a negative number is a _____ number.

c. A positive number multiplied by itself twice is a _____ number.

d. A negative number multiplied by itself twice is a _____ number.

4. **REASONING** Can a negative number have a cube root? Give an example to support your explanation.

5. **IN YOUR OWN WORDS** How is the cube root of a number different from the square root of a number?

6. Give an example of a number whose square root and cube root are equal.

7. A cube has a volume of 13,824 cubic meters. Use a calculator to find the edge length.

Practice Use what you learned about cube roots to complete Exercises 3–5 on page 298.

7.2 Lesson

Check It Out Lesson Tutorials BigIdeasMath.com

Key Vocabulary
cube root, *p. 296*
perfect cube, *p. 296*

A **cube root** of a number is a number that, when multiplied by itself, and then multiplied by itself again, equals the given number. A **perfect cube** is a number that can be written as the cube of an integer. The symbol $\sqrt[3]{\ }$ is used to represent a cube root.

EXAMPLE 1 Finding Cube Roots

Find each cube root.

a. $\sqrt[3]{8}$

∴ Because $2^3 = 8$, $\sqrt[3]{8} = \sqrt[3]{2^3} = 2$.

b. $\sqrt[3]{-27}$

∴ Because $(-3)^3 = -27$, $\sqrt[3]{-27} = \sqrt[3]{(-3)^3} = -3$.

c. $\sqrt[3]{\frac{1}{64}}$

∴ Because $\left(\frac{1}{4}\right)^3 = \frac{1}{64}$, $\sqrt[3]{\frac{1}{64}} = \sqrt[3]{\left(\frac{1}{4}\right)^3} = \frac{1}{4}$.

Cubing a number and finding a cube root are inverse operations. You can use this relationship to evaluate expressions and solve equations involving cubes.

EXAMPLE 2 Evaluating Expressions Involving Cube Roots

Evaluate each expression.

a. $2\sqrt[3]{-216} - 3 = 2(-6) - 3$ Evaluate the cube root.

$= -12 - 3$ Multiply.

$= -15$ Subtract.

b. $\left(\sqrt[3]{125}\right)^3 + 21 = 125 + 21$ Evaluate the power using inverse operations.

$= 146$ Add.

On Your Own

Now You're Ready
Exercises 6–17

Find the cube root.

1. $\sqrt[3]{1}$
2. $\sqrt[3]{-343}$
3. $\sqrt[3]{-\frac{27}{1000}}$

Evaluate the expression.

4. $18 - 4\sqrt[3]{8}$
5. $\left(\sqrt[3]{-64}\right)^3 + 43$
6. $5\sqrt[3]{512} - 19$

EXAMPLE 3 Evaluating an Algebraic Expression

Evaluate $\frac{x}{4} + \sqrt[3]{\frac{x}{3}}$ when $x = 192$.

$$\frac{x}{4} + \sqrt[3]{\frac{x}{3}} = \frac{192}{4} + \sqrt[3]{\frac{192}{3}}$$ Substitute 192 for x.

$$= 48 + \sqrt[3]{64}$$ Simplify.

$$= 48 + 4$$ Evaluate the cube root.

$$= 52$$ Add.

On Your Own

Now You're Ready
Exercises 18–20

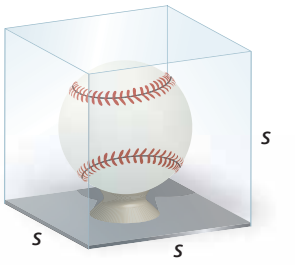
Evaluate the expression for the given value of the variable.

7. $\sqrt[3]{8y} + y, y = 64$ **8.** $2b - \sqrt[3]{9b}, b = -3$

EXAMPLE 4 Real-Life Application

Find the surface area of the baseball display case.

The baseball display case is in the shape of a cube. Use the formula for the volume of a cube to find the edge length s.



Volume = 125 in.^{3}

Remember

The volume V of a cube with edge length s is given by $V = s^3$. The surface area S is given by $S = 6s^2$.

$$V = s^3$$ Write formula for volume.

$$125 = s^3$$ Substitute 125 for V.

$$\sqrt[3]{125} = \sqrt[3]{s^3}$$ Take the cube root of each side.

$$5 = s$$ Simplify.

The edge length is 5 inches. Use a formula to find the surface area of the cube.

$$S = 6s^2$$ Write formula for surface area.

$$= 6(5)^2$$ Substitute 5 for s.

$$= 150$$ Simplify.

∴ So, the surface area of the baseball display case is 150 square inches.

On Your Own

9. The volume of a music box that is shaped like a cube is 512 cubic centimeters. Find the surface area of the music box.

7.2 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

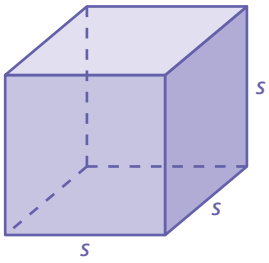
Vocabulary and Concept Check

1. **VOCABULARY** Is 25 a perfect cube? Explain.
2. **REASONING** Can the cube of an integer be a negative number? Explain.

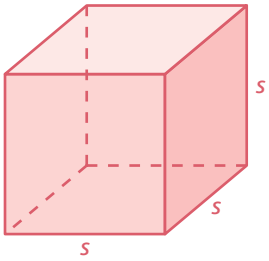
Practice and Problem Solving

Find the edge length of the cube.

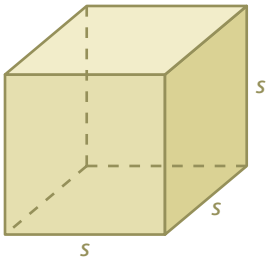
3. Volume = 125,000 in.^{3}



4. Volume = $\frac{1}{27}$ ft^3



5. Volume = 0.064 m^3



Find the cube root.

6. $\sqrt[3]{729}$
7. $\sqrt[3]{-125}$
8. $\sqrt[3]{-1000}$
9. $\sqrt[3]{1728}$
10. $\sqrt[3]{-\frac{1}{512}}$
11. $\sqrt[3]{\frac{343}{64}}$

Evaluate the expression.

12. $18 - \left(\sqrt[3]{27}\right)^3$
13. $\left(\sqrt[3]{-\frac{1}{8}}\right)^3 + 3\frac{3}{4}$
14. $5\sqrt[3]{729} - 24$
15. $\frac{1}{4} - 2\sqrt[3]{-\frac{1}{216}}$
16. $54 + \sqrt[3]{-4096}$
17. $4\sqrt[3]{8000} - 6$

Evaluate the expression for the given value of the variable.

18. $\sqrt[3]{\frac{n}{4}} + \frac{n}{10}$, $n = 500$
19. $\sqrt[3]{6w} - w$, $w = 288$
20. $2d + \sqrt[3]{-45d}$, $d = 75$

21. **STORAGE CUBE** The volume of a plastic storage cube is 27,000 cubic centimeters. What is the edge length of the storage cube?

22. **ICE SCULPTURE** The volume of a cube of ice for an ice sculpture is 64,000 cubic inches.
    a. What is the edge length of the cube of ice?
    b. What is the surface area of the cube of ice?

Copy and complete the statement with <, >, or =.

23. $-\frac{1}{4} \square \sqrt[3]{-\frac{8}{125}}$ **24.** $\sqrt[3]{0.001} \square 0.01$ **25.** $\sqrt[3]{64} \square \sqrt{64}$

26. DRAG RACE The estimated velocity v (in miles per hour) of a car at the end of a drag race is $v = 234\sqrt[3]{\frac{p}{w}}$, where p is the horsepower of the car and w is the weight (in pounds) of the car. A car has a horsepower of 1311 and weighs 2744 pounds. Find the velocity of the car at the end of a drag race. Round your answer to the nearest whole number.

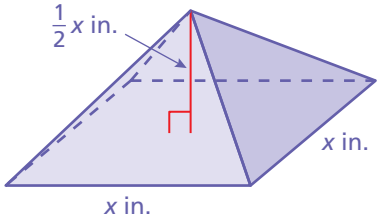
27. NUMBER SENSE There are three numbers that are their own cube roots. What are the numbers?

28. LOGIC Each statement below is true for square roots. Determine whether the statement is also true for cube roots. Explain your reasoning and give an example to support your explanation.

a. You cannot find the square root of a negative number.

b. Every positive number has a positive square root and a negative square root.

29. GEOMETRY The pyramid has a volume of 972 cubic inches. What are the dimensions of the pyramid?



30. RATIOS The ratio $125 : x$ is equivalent to the ratio $x^2 : 125$. What is the value of x?

Critical Thinking **Solve the equation.**

31. $(3x + 4)^3 = 2197$ **32.** $(8x^3 - 9)^3 = 5832$ **33.** $\left((5x - 16)^3 - 4\right)^3 = 216{,}000$

Fair Game Review What you learned in previous grades & lessons

Evaluate the expression. *(Skills Review Handbook)*

34. $3^2 + 4^2$ **35.** $8^2 + 15^2$ **36.** $13^2 - 5^2$ **37.** $25^2 - 24^2$

38. MULTIPLE CHOICE Which linear function is shown by the table? *(Section 6.3)*

x	1	2	3	4
y	4	7	10	13

A $y = \frac{1}{3}x + 1$ **B** $y = 4x$ **C** $y = 3x + 1$ **D** $y = \frac{1}{4}x$