8.1 Volumes of Cylinders

Essential Question How can you find the volume of a cylinder?

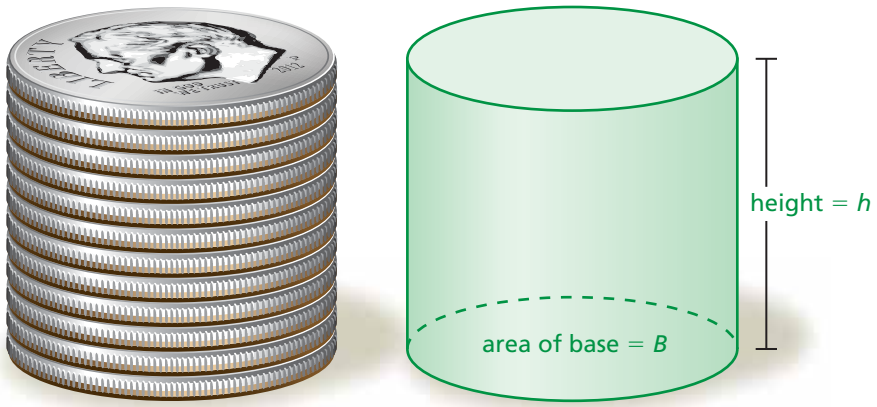
1 ACTIVITY: Finding a Formula Experimentally

Work with a partner.

a. Find the area of the face of a coin.

b. Find the volume of a stack of a dozen coins.

c. Write a formula for the volume of a cylinder.



2 ACTIVITY: Making a Business Plan

Work with a partner. You are planning to make and sell three different sizes of cylindrical candles. You buy 1 cubic foot of candle wax for $20 to make 8 candles of each size.

a. Design the candles. What are the dimensions of each size of candle?

b. You want to make a profit of $100. Decide on a price for each size of candle.

c. Did you set the prices so that they are proportional to the volume of each size of candle? Why or why not?

Geometry

In this lesson, you will

- find the volumes of cylinders.
- find the heights of cylinders given the volumes.
- solve real-life problems.

3 ACTIVITY: Science Experiment

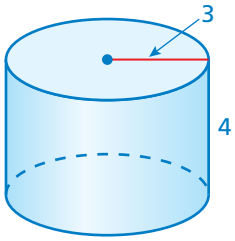
Work with a partner. Use the diagram to describe how you can find the volume of a small object.

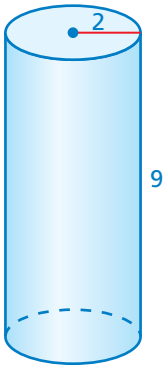
4 ACTIVITY: Comparing Cylinders

Work with a partner.

a. Just by looking at the two cylinders, which one do you think has the greater volume? Explain your reasoning.

b. Find the volume of each cylinder. Was your prediction in part (a) correct? Explain your reasoning.





Math Practice

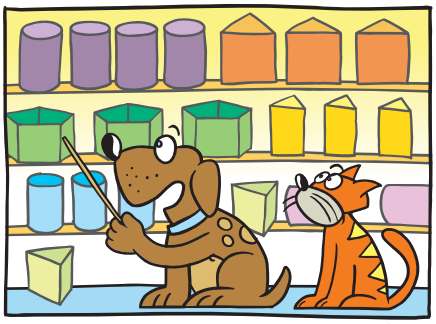
Consider Similar Problems

How can you use the results of Activity 1 to find the volumes of the cylinders?

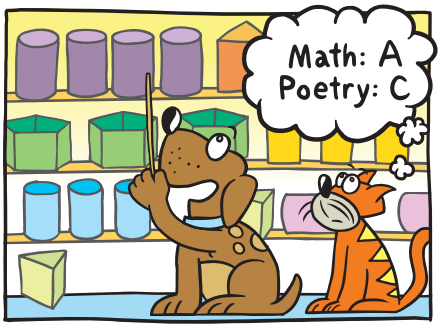
What Is Your Answer?

5. IN YOUR OWN WORDS How can you find the volume of a cylinder?

6. Compare your formula for the volume of a cylinder with the formula for the volume of a prism. How are they the same?



"Here's how I remember how to find the volume of any prism or cylinder."



"Base times tall, will fill 'em all."

Practice Use what you learned about the volumes of cylinders to complete Exercises 3–5 on page 338.

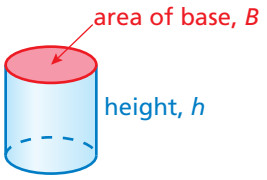
8.1 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Idea

Volume of a Cylinder

Words The volume V of a cylinder is the product of the area of the base and the height of the cylinder.



Algebra $V = Bh$

Area of base — Height of cylinder

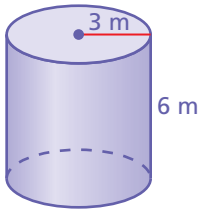
EXAMPLE 1 Finding the Volume of a Cylinder

Find the volume of the cylinder. Round your answer to the nearest tenth.

$V = Bh$ Write formula for volume.

$= \pi(3)^2(6)$ Substitute.

$= 54\pi \approx 169.6$ Use a calculator.



∴ The volume is about 169.6 cubic meters.

Study Tip

Because $B = \pi r^2$, you can use $V = \pi r^2 h$ to find the volume of a cylinder.

EXAMPLE 2 Finding the Height of a Cylinder

Find the height of the cylinder. Round your answer to the nearest whole number.

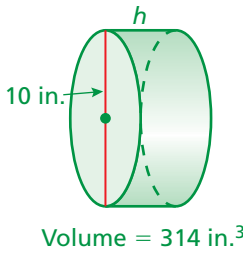
The diameter is 10 inches. So, the radius is 5 inches.

$V = Bh$ Write formula for volume.

$314 = \pi(5)^2(h)$ Substitute.

$314 = 25\pi h$ Simplify.

$4 \approx h$ Divide each side by 25π.



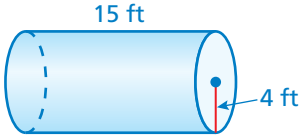
∴ The height is about 4 inches.

On Your Own

Now You're Ready
Exercises 3–11 and 13–15

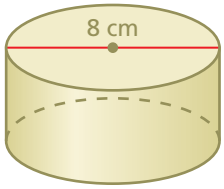
Find the volume V or height h of the cylinder. Round your answer to the nearest tenth.

1.



$V \approx$ ____

2.



$h \approx$ ____

EXAMPLE 3 Real-Life Application

STEM VIDEO

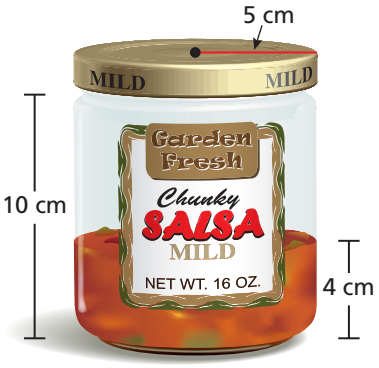
How much salsa is missing from the jar?

The empty space in the jar is a cylinder with a height of $10 - 4 = 6$ centimeters and a radius of 5 centimeters.

$V = Bh$ Write formula for volume.

$= \pi(5)^2(6)$ Substitute.

$= 150\pi \approx 471$ Use a calculator.



So, about 471 cubic centimeters of salsa are missing from the jar.

EXAMPLE 4 Real-Life Application

About how many gallons of water does the watercooler bottle contain? ($1 \text{ ft}^3 \approx 7.5$ gal)

(A) 5.3 gallons (B) 10 gallons (C) 17 gallons (D) 40 gallons

Find the volume of the cylinder. The diameter is 1 foot. So, the radius is 0.5 foot.

$V = Bh$ Write formula for volume.

$= \pi(0.5)^2(1.7)$ Substitute.

$= 0.425\pi \approx 1.3352$ Use a calculator.

So, the bottle contains about 1.3352 cubic feet of water. To find the number of gallons it contains, multiply by the conversion factor $\frac{7.5 \text{ gal}}{1 \text{ ft}^3}$.

$$1.3352 \text{ ft}^3 \times \frac{7.5 \text{ gal}}{1 \text{ ft}^3} \approx 10 \text{ gal}$$

The watercooler bottle contains about 10 gallons of water. So, the correct answer is (B).

On Your Own

Now You're Ready
Exercise 12

3. **WHAT IF?** In Example 3, the height of the salsa in the jar is 5 centimeters. How much salsa is missing from the jar?
4. A cylindrical water tower has a diameter of 15 meters and a height of 5 meters. About how many gallons of water can the tower contain? ($1 \text{ m}^3 \approx 264$ gal)

8.1 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

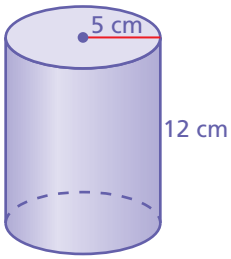
1. DIFFERENT WORDS, SAME QUESTION Which is different? Find "both" answers.

How much does it take to fill the cylinder?

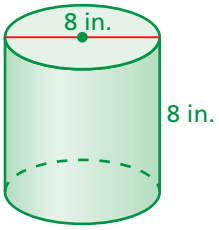
What is the capacity of the cylinder?

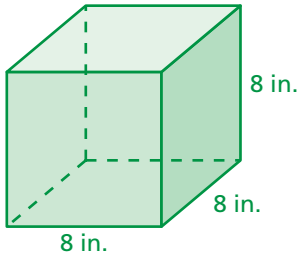
How much does it take to cover the cylinder?

How much does the cylinder contain?



2. REASONING Without calculating, which of the solids has the greater volume? Explain.



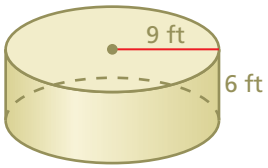


Practice and Problem Solving

Find the volume of the cylinder. Round your answer to the nearest tenth.

1

3.



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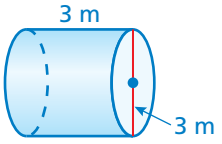

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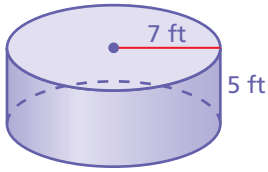

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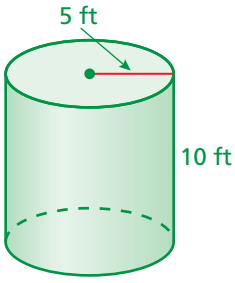

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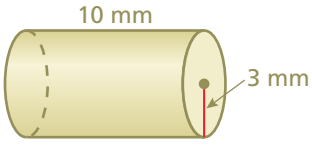

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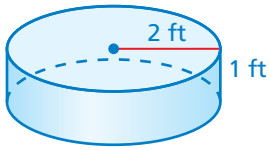

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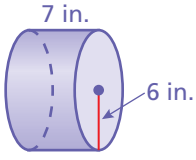

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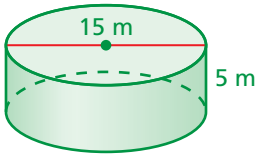

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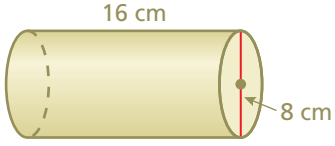
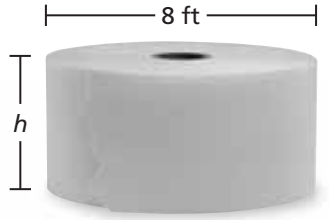


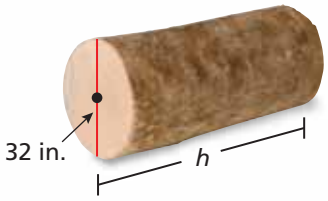
4 **12. SWIMMING POOL** A cylindrical swimming pool has a diameter of 16 feet and a height of 4 feet. About how many gallons of water can the pool contain? Round your answer to the nearest whole number. ($1 \text{ ft}^3 \approx 7.5$ gal)

Find the missing dimension of the cylinder. Round your answer to the nearest whole number.

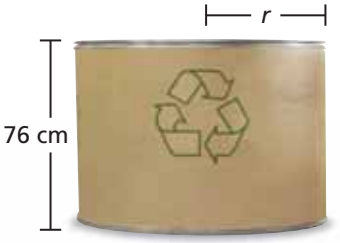
13. Volume = 250 ft^3



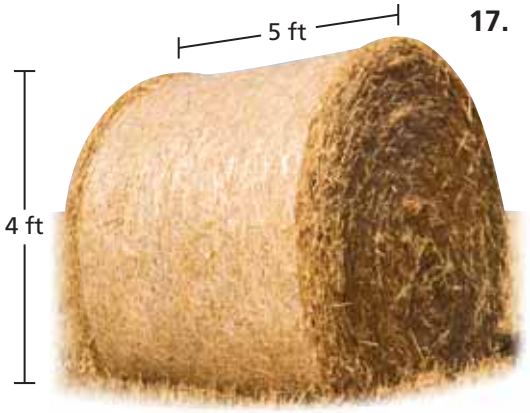
14. Volume = $10{,}000\pi$ $in.^3$



15. Volume = 600,000 cm^3



16. CRITICAL THINKING How does the volume of a cylinder change when its diameter is halved? Explain.



Round hay bale

17. MODELING A traditional "square" bale of hay is actually in the shape of a rectangular prism. Its dimensions are 2 feet by 2 feet by 4 feet. How many square bales contain the same amount of hay as one large "round" bale?

18. ROAD ROLLER A tank on a road roller is filled with water to make the roller heavy. The tank is a cylinder that has a height of 6 feet and a radius of 2 feet. One cubic foot of water weighs 62.5 pounds. Find the weight of the water in the tank.

19. VOLUME A cylinder has a surface area of 1850 square meters and a radius of 9 meters. Estimate the volume of the cylinder to the nearest whole number.

20. Problem Solving Water flows at 2 feet per second through a pipe with a diameter of 8 inches. A cylindrical tank with a diameter of 15 feet and a height of 6 feet collects the water.

a. What is the volume, in cubic inches, of water flowing out of the pipe every second?

b. What is the height, in inches, of the water in the tank after 5 minutes?

c. How many minutes will it take to fill 75% of the tank?

Fair Game Review What you learned in previous grades & lessons

Tell whether the triangle with the given side lengths is a right triangle. *(Section 7.5)*

21. 20 m, 21 m, 29 m **22.** 1 in., 2.4 in., 2.6 in. **23.** 5.6 ft, 8 ft, 10.6 ft

24. MULTIPLE CHOICE Which ordered pair is the solution of the linear system $3x + 4y = -10$ and $2x - 4y = 0$? *(Section 5.3)*

(A) $(-6, 2)$ (B) $(2, -6)$ (C) $(-2, -1)$ (D) $(-1, -2)$