2.5 THE REMAINDER THEOREM & FACTOR THEOREM

OBJECTIVE: To be able to divide polynomials and relate the result to the remainder theorem and factor theorem.

*USING LONG DIVISION TO DIVIDE POLYNOMIALS:

EXAMPLE #1: Divide.

a) $(4x^3 + 2x^2 - 2x + 6) \div (2x - 3)$

$$\begin{array}{r} 2x^2 + 4x + 5 + \frac{21}{2x-3} \\ 2x-3 \,\big)\, 4x^3 + 2x^2 - 2x + 6 \\ -4x^3 + 6x^2 \qquad\qquad \\ \hline 8x^2 - 2x \qquad \\ -8x^2 + 12x \qquad \\ \hline 10x + 6 \\ -10x + 15 \\ \hline 21 \end{array}$$

$\frac{4x^3}{2x} = 2x^2$

$\frac{8x^2}{2x} = 4x$

$\frac{10x}{2x} = 5$

b) $(2x^4 + 3x^3 + 5x + 1) \div (x^2 - 2x + 2)$

$$\begin{array}{r} 2x^2 + 7x + 10 + \frac{11x-9}{x^2-2x+2} \\ x^2-2x+2 \,\big)\, 2x^4 + 3x^3 + 0x^2 + 5x + 1 \\ -2x^4 + 4x^3 \mp 4x^2 \qquad\qquad \\ \hline 7x^3 - 4x^2 + 5x \qquad \\ -7x^3 \pm 14x^2 \mp 14x \qquad \\ \hline 10x^2 - 9x + 1 \\ -10x^2 \pm 20x \mp 20 \\ \hline 11x - 9 \end{array}$$

$\frac{2x^4}{x^2} = 2x^2$

$\frac{7x^3}{x^2} = 7x$

$\frac{10x^2}{x^2} = 10$

Investigating Polynomial Division

#1: Divide $f(x) = 3x^3 - 2x^2 + 2x - 5$ by $x - 2$. What is the quotient? What is the remainder?

$$
\begin{array}{r l}
 & 3x^2 + 4x + 10 \quad + \frac{15}{x-2} \\
x-2 \,) & 3x^3 - 2x^2 + 2x - 5 \\
 & -3x^3 + 6x^2 \\
 & \quad 4x^2 + 2x \\
 & \quad -4x^2 + 8x \\
 & \qquad 10x - 5 \\
 & \qquad -10x + 20 \\
 & \qquad\qquad 15
\end{array}
$$

#2: Use synthetic substitution to evaluate f(2). How is f(2) related to the remainder?

$$
\begin{array}{c|cccc}
2 & 3 & -2 & 2 & -5 \\
 & \downarrow & 6 & 8 & 20 \\
\hline
 & 3 & 4 & 10 & 15 \\
 & x^2 & x & \text{Constant} & R
\end{array}
$$

$f(2) = 15$

↑remainder

#3: What do you notice about the other numbers in the last row?

They are the coefficients of the answer

$\underline{3}x^2 + \underline{4}x + \underline{10}$

**THE REMAINDER THEOREM: If a polynomial is divided by x – k, then the remainder is r = f(k).

*USING SYNTHETIC DIVISION:

EXAMPLE #2: Divide.

a) $(x^3 + 2x^2 - 6x - 9) \div (x - 2)$

$$\begin{array}{r|rrrr} 2 & 1 & 2 & -6 & -9 \\ & \downarrow & 2 & 8 & 4 \\ \hline & 1 & 4 & 2 & \boxed{-5} \\ & x^2 & x & c & R \end{array}$$

$$\boxed{x^2+4x+2+\frac{-5}{x-2}}$$

b) $(2x^3 + 3x + 5) \div (x - 1)$

$$\begin{array}{r|rrrr} 1 & 2 & 0 & 3 & 5 \\ & \downarrow & 2 & 2 & 5 \\ \hline & 2 & 2 & 5 & \boxed{10} \\ & x^2 & x & c & R \end{array}$$

$$\boxed{2x^2+2x+5+\frac{10}{x-1}}$$

c) $(x^3 + 3x^2 + 3x + 9) \div (x + 3)$

$$\begin{array}{r|rrrr} -3 & 1 & 3 & 3 & 9 \\ & \downarrow & -3 & 0 & -9 \\ \hline & 1 & 0 & 3 & \boxed{0} \\ & x^2 & x & c & R \end{array}$$

$$\boxed{x^2+3}$$

*THE FACTOR THEOREM: A polynomial f(x) has a factor x – k if and only if f(k) = 0.

→The number *k* is called a ZERO of the function because the remainder is 0.

EXAMPLE #3: Factoring a Polynomial.

$(x+3)$

a) Factor $f(x) = 2x^3 + 11x^2 + 18x + 9$; given that $f(-3) = 0$.

-3	2	11	18	9
	↓	-6	-15	-9
	2	5	3	0

$2x^2+5x+3$

6
2 3

$2x^2+2x+3x+3$

$2x(x+1)+3(x+1)$

$(2x+3)(x+1)$

$x+4$

b) Factor $f(x) = 3x^3 + 13x^2 + 2x - 8$; given that $f(-4) = 0$.

-4	3	13	2	-8
	↓	-12	-4	8
	3	1	-2	0

$3x^2+x-2$

AC
6

$3x^2+3x+2x-2$

$3x(x+1)+2(x+1)$

$(3x+2)(x+1)$

$x-3$

c) Factor $f(x) = 6x^3 - 25x^2 + 16x + 15$; given that $f(3) = 0$.

3	6	-25	16	15
	↓	18	-21	-15
	6	-7	-5	0

$6x^2-7x-5$

2
3
30 5

$6x^2-10x+3x-5$

$2x(3x-5)+1(3x-5)$

$(2x+1)(3x-5)$

*FINDING THE ZEROS OF A POLYNOMIAL FUNCTION:

EXAMPLE #4: Given one zero, find the other zeros of the function.

a) $f(x) = x^3 + 6x^2 + 3x - 10$; $x = -5$

$$
\begin{array}{r|rrrr}
-5 & 1 & 6 & 3 & -10 \\
 & \downarrow & -5 & -5 & 10 \\
\hline
 & 1 & 1 & -2 & 0
\end{array}
$$

$x^2+x-2=0$

$(x+2)(x-1)=0$

$x=-2,1$

b) $f(x) = 2x^3 - 9x^2 - 32x - 21$; given that $x = 7$

$$
\begin{array}{r|rrrr}
7 & 2 & -9 & -32 & -21 \\
 & \downarrow & 14 & 35 & 21 \\
\hline
 & 2 & 5 & 3 & 0
\end{array}
$$

$2x^2+5x+3=0$ 6

$2x^2+2x+3x+3$

$2x(x+1)3(x+1)$

$(2x+3)(x+1)=0$

$x=-\frac{3}{2},-1$

c) $f(x) = x^3 - 2x^2 - 9x + 18$; given that $x = 2$

$$
\begin{array}{r|rrrr}
2 & 1 & -2 & -9 & 18 \\
 & \downarrow & 2 & 0 & -18 \\
\hline
 & 1 & 0 & -9 & 0
\end{array}
$$

x^2-9

$(x+3)(x-3)$

$x=-3,3$

d) $f(x)=2x^3-10x^2-11x-9$; 9

$$
\begin{array}{r|rrrr}
9 & 2 & -10 & -71 & -9 \\
 & \downarrow & 18 & 72 & 9 \\
\hline
 & 2 & 8 & 1 & 0
\end{array}
$$

$2x^2+8x+1$

$$\frac{-8 \pm \sqrt{64-4(2)(1)}}{4}$$

$$\frac{-8 \pm \sqrt{56}}{4}$$

$$\frac{-8 \pm 2\sqrt{14}}{4}$$

$$\frac{-4 \pm \sqrt{14}}{2}$$