

5.5 Review

1.) a) $\cos u = \frac{4}{5} \rightarrow Q 4$ $\sin u = -\frac{3}{5}$ $\tan u = -\frac{3}{4}$

$$\begin{aligned}\sin 2u &= 2 \cos u \sin u \\ &= 2 \left(\frac{4}{5} \right) \left(-\frac{3}{5} \right) \\ &= \boxed{-\frac{24}{25}}\end{aligned}$$

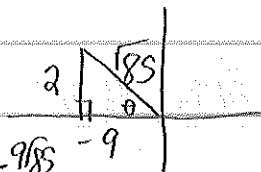
$$\begin{aligned}\cos 2u &= \cos^2 u - \sin^2 u \\ &= \left(\frac{4}{5} \right)^2 - \left(-\frac{3}{5} \right)^2 \\ &= \frac{16}{25} - \frac{9}{25} \\ &= \boxed{\frac{7}{25}}\end{aligned}$$

$$\tan 2u = \boxed{-\frac{24}{7}}$$

b) $\tan u = -\frac{2}{9}$ Q 2

$$\sin u = \frac{2\sqrt{85}}{85}$$

$$\cos u = -\frac{9\sqrt{85}}{85}$$



$$\begin{aligned}4 + 81 &= c^2 \\ 85 &= c^2 \\ \sqrt{85} &= c\end{aligned}$$

$$\begin{aligned}\sin 2u &= 2 \cos u \sin u \\ &= 2 \left(-\frac{9\sqrt{85}}{85} \right) \left(\frac{2\sqrt{85}}{85} \right) \\ &= \frac{-36(85)}{85(85)} = \boxed{-\frac{36}{85}}\end{aligned}$$

$$\begin{aligned}\cos 2u &= 2 \cos^2 u - 1 \\ &= 2 \left(-\frac{9\sqrt{85}}{85} \right)^2 - 1 \\ &= 2 \left(\frac{81 \cdot 85}{85 \cdot 85} \right) - 1\end{aligned}$$

$$= \frac{162}{85} - \frac{85}{85} = \boxed{\frac{77}{85}}$$

$$\tan 2u = \boxed{-\frac{36}{77}}$$

c) $\sin u = \frac{12}{13}$, $\frac{\pi}{2} < u < \pi$ $\cos u = -\frac{5}{13}$, $\tan u = -\frac{12}{5}$

$$\begin{aligned}\sin 2u &= 2 \cos u \sin u \\ &= 2 \left(-\frac{5}{13} \cdot \frac{12}{13} \right) \\ &= \boxed{-\frac{120}{169}}\end{aligned}$$

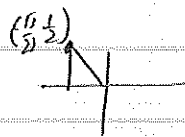
$$\begin{aligned}\cos 2u &= 2 \cos^2 u - 1 \\ &= 2 \left(-\frac{5}{13} \right)^2 - 1 \\ &= 2 \left(\frac{25}{169} \right) - 1 \\ &= \frac{50}{169} - \frac{169}{169} = \boxed{-\frac{119}{169}}\end{aligned}$$

$$\tan 2u = \boxed{\frac{120}{119}}$$

$$2. a) \cos 120 \quad 2u = 120 \quad \sin u = \frac{\sqrt{3}}{2} \quad \cos u = \frac{1}{2}$$

$$u = 60$$

$$\begin{aligned} \cos 2(60) &= 2 \cos^2 u - 1 \\ &= 2 \left(\frac{1}{2}\right)^2 - 1 \\ &= 2 \left(\frac{1}{4}\right) - 1 \\ &= \frac{1}{2} - 1 = \boxed{-\frac{1}{2}} \end{aligned}$$



$$b) \tan 300 \quad 2u = 300 \quad \sin u = \frac{1}{2} \quad \cos u = -\frac{\sqrt{3}}{2}$$

$$u = 150$$

$$\tan u = -\frac{\sqrt{3}}{3}$$

$$\begin{aligned} \tan 2(150) &= \frac{2 \tan u}{1 - \tan^2 u} = \frac{2 \left(-\frac{\sqrt{3}}{3}\right)}{1 - \left(\frac{\sqrt{3}}{3}\right)^2} = \frac{-\frac{2\sqrt{3}}{3}}{1 - \frac{3}{9}} = \frac{-\frac{2\sqrt{3}}{3}}{\frac{3}{3} - \frac{1}{3}} \\ &= \frac{-\frac{2\sqrt{3}}{3} \cdot \frac{3}{2}}{\frac{2}{3}} = \frac{-2\sqrt{3}}{2} = -\sqrt{3} \end{aligned}$$

$$c) \sin 90 \quad 2u = 90 \quad \sin u = \frac{\sqrt{2}}{2} \quad \cos u = \frac{\sqrt{2}}{2}$$

$$u = 45$$

$$\begin{aligned} \sin 2(45) &= 2 \sin u \cos u \\ &= 2 \left(\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2}\right) \\ &= \frac{4}{4} = 1 \end{aligned}$$

$\sin$	$\frac{4}{5}$	+
$\cos$	$\frac{3}{5}$	-
$\tan$	$\frac{4}{3}$	-

$$\begin{array}{r} 49 \\ - 4 \\ \hline 45 \end{array}$$

$$\cos \frac{u}{2} = \sqrt{\frac{1 + \cos u}{2}}$$

$$= \sqrt{1 - \frac{2}{7}}$$

$$= \sqrt{\frac{7-2}{7}} = \sqrt{\frac{5}{7}} \approx 0.845$$

$$= \sqrt{\frac{5}{14}} = \frac{\sqrt{5}}{\sqrt{14}} = \frac{\sqrt{70}}{14}$$

$$\tan \frac{y}{2} = \frac{3\sqrt{4}}{\sqrt{70}} = \frac{3\sqrt{4}}{\sqrt{4 \cdot 5}} = \frac{3}{\sqrt{5}} = \boxed{\frac{3\sqrt{5}}{5}}$$

c) $\sec u = -6$ Q2 $\rightarrow \frac{u}{2}$ in Q1 $\frac{u}{2}$ all +

$$\cos u = -\frac{1}{6}$$

$$\sin \frac{\alpha}{2} = \frac{1 + \frac{1}{p}}{2}$$

$$\cos \frac{\gamma}{2} = \sqrt{\frac{1 - \frac{1}{b}}{2}}$$

$$\begin{array}{r} 5 \frac{1}{2} \\ 2 \overline{) 11} \\ \underline{10} \\ 1 \end{array}$$

$$\tan \frac{\gamma}{2} = \frac{\sqrt{21} \cdot \sqrt{5}}{3 \cdot 1} = \frac{\sqrt{105}}{3} = \frac{1}{3} \sqrt{105} = \sqrt{\frac{105}{9}} = \sqrt{\frac{7}{12}} = \frac{\sqrt{7}}{\sqrt{12}} = \frac{\sqrt{7}}{2\sqrt{3}} = \frac{\sqrt{84}}{12} = \frac{\sqrt{61}}{6}$$

$$= \sqrt{\frac{1}{\frac{5}{12}}} = \frac{\sqrt{5} \cdot \sqrt{12}}{\sqrt{12} \cdot \sqrt{12}} = \frac{\sqrt{60} = \sqrt{15}}{12}$$

$$\cos \frac{\psi}{2} = \left| \frac{\sqrt{15}}{6} \right|$$

$$\cos \frac{\nu}{2} = \frac{\sqrt{15}}{6}$$

$$\sin \frac{\theta}{2} = \frac{|a|}{b}$$

Q1 +

$$4) a) \sin 15$$

$$\frac{y}{2} = 15$$

$$u = 30$$

$$\sin u = \frac{1}{2}, \cos u = \frac{\sqrt{3}}{2}$$

$$\sin \frac{y}{2} = \sqrt{\frac{1 - \cos u}{2}}$$

$$= \sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}}$$

$$= \sqrt{\frac{\frac{2}{2} - \frac{\sqrt{3}}{2}}{2}}$$

$$= \sqrt{\frac{\frac{2 - \sqrt{3}}{2} \cdot \frac{1}{2}}{1}}$$

$$= \sqrt{\frac{2 - \sqrt{3}}{4}} = \boxed{\frac{\sqrt{2 - \sqrt{3}}}{2}}$$

$$b) \cos 112^{\circ} 30'$$

$$\frac{y}{2} = 112.5$$

$$u = 225$$

Q2 cos is neg

$$\cos u = -\frac{\sqrt{2}}{2}$$

$$\cos \frac{y}{2} = -\sqrt{\frac{1 + \cos u}{2}}$$

$$= -\sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{2}}$$

$$= -\sqrt{\frac{\frac{2 - \sqrt{2}}{2} \cdot \frac{1}{2}}{1}}$$

$$= -\sqrt{\frac{2 - \sqrt{2}}{4}}$$

$$= -\frac{\sqrt{2 - \sqrt{2}}}{2}$$

$$4. c \tan \frac{7\pi}{8} \quad 2 \cdot \frac{u}{2} = \frac{7\pi}{84} \cdot 2$$

$$u = \frac{7\pi}{4} = \frac{7}{4}\pi$$

$$\sin u = \frac{-\sqrt{2}}{2}$$

$$\cos u = \frac{\sqrt{2}}{2}$$

$$\tan \frac{7\pi}{4} = -1$$

$$\tan \frac{u}{2} = \frac{1 - \cos u}{\sin u}$$

$$= \frac{1 - \frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = \frac{\frac{2}{2} - \frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = \frac{2 - \sqrt{2}}{-\sqrt{2}} \cdot \frac{2}{2} = \frac{-2 + \sqrt{2}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{-2\sqrt{2} + 2}{2} = \boxed{-\sqrt{2} + 1}$$

$$\text{Int: } [0, 2\pi)$$

$$5) a) 2 \sin 2x - \sqrt{2} = 0$$

$$\frac{2 \sin u}{2} = \frac{\sqrt{2}}{2}$$

$$\sin u = \frac{\sqrt{2}}{2}$$

$$u = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

$$\frac{\pi}{4} + \frac{8\pi}{4} = \frac{9\pi}{4} \quad \frac{3\pi}{4} + \frac{8\pi}{4} = \frac{11\pi}{4}$$

$$u = 2x \text{ double } <$$

$$\text{int: } [0, 4\pi)$$

$\frac{1}{2} \cdot 2x = \frac{\pi}{4} \cdot \frac{1}{2}$	$x = \frac{\pi}{8}$
$\frac{1}{2} \cdot 2x = \frac{3\pi}{4} \cdot \frac{1}{2}$	$x = \frac{3\pi}{8}$
$\frac{1}{2} \cdot 2x = \frac{5\pi}{4} \cdot \frac{1}{2}$	$x = \frac{5\pi}{8}$
$\frac{1}{2} \cdot 2x = \frac{7\pi}{4} \cdot \frac{1}{2}$	$x = \frac{7\pi}{8}$

$$b) 4 \cos^2 2x - 3 = 0$$

$$\frac{4 \cos^2 u}{4} = \frac{3}{4}$$

$$\cos^2 u = \frac{3}{4}$$

$$\cos u = \pm \frac{\sqrt{3}}{2}$$

$$u = 2x \text{ double } < \text{ int: } [0, 4\pi)$$

$$\frac{11\pi}{6} = 2\pi + \frac{5\pi}{6}$$

$$\frac{19\pi}{6} = 2\pi + \frac{7\pi}{6}$$

$$\frac{\pi}{6} + 2\pi = \frac{13\pi}{6}$$

$$\frac{11\pi}{6} + 2\pi = \frac{23\pi}{6}$$

$$\frac{1}{2} \cdot 2x = \frac{\pi}{6} \cdot \frac{1}{2}$$

$$\frac{1}{2} \cdot 2x = \frac{5\pi}{6} \cdot \frac{1}{2}$$

$$\frac{1}{2} \cdot 2x = \frac{7\pi}{6} \cdot \frac{1}{2}$$

$$\frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$$

5c) $\sin 2x \sin x = \cos x$

$$2 \sin x \cos x \sin x = \cos x$$

$$2 \sin^2 x \cos x = \cos x$$

$$- \cos x - \cos x$$

$$2 \sin^2 x \cos x - \cos x = 0$$

$$\cos x (2 \sin^2 x - 1) = 0$$

$$\cos x = 0$$

$$\frac{\pi}{2}, \frac{3\pi}{2}$$

$$2 \sin^2 x - 1 = 0$$

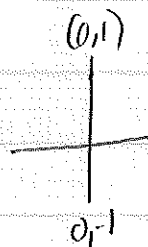
$$\frac{2 \sin^2 x}{2} = \frac{1}{2}$$

$$\sin^2 x = \frac{1}{2}$$

$$\sin x = \pm \frac{1}{\sqrt{2}}$$

$$\sin x = \pm \frac{\sqrt{2}}{2}$$

$$\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$



5d) $2 \cos x + \sin 2x = 0$

$$2 \cos x + 2 \sin x \cos x = 0$$

$$2 \cos x (1 + \sin x) = 0$$

$$2 \cos x = 0$$

$$\cos x = 0$$

$$\frac{\pi}{2}, \frac{3\pi}{2}$$

$$1 + \sin x = 0$$

$$\sin x = -1$$

$$\frac{3\pi}{2}$$

$$6) a) 6 \sin x \cos x = 3 \sin 2x$$

$$3(2 \sin x \cos x)$$

$$3 \sin 2x = 3 \sin 2x \quad \checkmark$$

$$6 \sin x \cos x = 3 \sin 2x$$

$$= 3(2 \sin x \cos x)$$

$$6 \sin x \cos x = 6 \sin x \cos x \quad \checkmark$$

$$b) \csc 2x = \frac{\csc x}{2 \cos x}$$

$$\frac{1}{\sin 2x}$$

$$\frac{1}{2 \sin x \cos x}$$

$$\frac{\csc x}{2 \cos x} = \frac{\csc x}{2 \cos x} \quad \checkmark$$

$$c) \cos^4 x - \sin^4 x = \cos 2x$$

$$(\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)$$

$$\cos 2x \quad 1$$

$$\cos 2x = \cos 2x$$

$$\checkmark$$

