

Summer Review Packet for Students Entering Calculus (all levels)

Complex Fractions

When simplifying complex fractions, multiply by a fraction equal to 1 which has a numerator and denominator composed of the common denominator of all the denominators in the complex fraction.

Example:

$$\frac{-7 - \frac{6}{x+1}}{\frac{5}{x+1}} = \frac{-7 - \frac{6}{x+1}}{\frac{5}{x+1}} \cdot \frac{x+1}{x+1} = \frac{-7x - 7 - 6}{5} = \frac{-7x - 13}{5}$$

$$\frac{\frac{-2}{x} + \frac{3x}{x-4}}{5 - \frac{1}{x-4}} = \frac{\frac{-2}{x} + \frac{3x}{x-4}}{5 - \frac{1}{x-4}} \cdot \frac{x(x-4)}{x(x-4)} = \frac{-2(x-4) + 3x(x)}{5(x)(x-4) - 1(x)} = \frac{-2x + 8 + 3x^2}{5x^2 - 20x - x} = \frac{3x^2 - 2x + 8}{5x^2 - 21x}$$

Simplify each of the following.

1. $\frac{\frac{25}{a} - a}{5 + a}$

2. $\frac{2 - \frac{4}{x+2}}{5 + \frac{10}{x+2}}$

3. $\frac{4 - \frac{12}{2x-3}}{5 + \frac{15}{2x-3}}$

4. $\frac{\frac{x}{x+1} - \frac{1}{x}}{\frac{x}{x+1} + \frac{1}{x}}$

5. $\frac{1 - \frac{2x}{3x-4}}{x + \frac{32}{3x-4}}$

Functions

To evaluate a function for a given value, simply plug the value into the function for x .

Recall: $(f \circ g)(x) = f(g(x))$ OR $f[g(x)]$ read “ f of g of x ” Means to plug the inside function (in this case $g(x)$) in for x in the outside function (in this case, $f(x)$).

Example: Given $f(x) = 2x^2 + 1$ and $g(x) = x - 4$ find $f(g(x))$.

$$\begin{aligned}f(g(x)) &= f(x - 4) \\ &= 2(x - 4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33\end{aligned}$$

Let $f(x) = 2x + 1$ and $g(x) = 2x^2 - 1$. Find each.

6. $f(2) =$ _____ 7. $g(-3) =$ _____ 8. $f(t+1) =$ _____

9. $f[g(-2)] =$ _____ 10. $g[f(m+2)] =$ _____ 11. $\frac{f(x+h) - f(x)}{h} =$ _____

Let $f(x) = \sin x$ Find each exactly.

12. $f\left(\frac{\pi}{2}\right) =$ _____ 13. $f\left(\frac{2\pi}{3}\right) =$ _____

Let $f(x) = x^2$, $g(x) = 2x + 5$, and $h(x) = x^2 - 1$. Find each.

14. $h[f(-2)] =$ _____ 15. $f[g(x-1)] =$ _____ 16. $g[h(x^3)] =$ _____

Find $\frac{f(x+h)-f(x)}{h}$ for the given function f .

17. $f(x) = 9x + 3$

18. $f(x) = 5 - 2x$

Intercepts and Points of Intersection

To find the x-intercepts, let $y = 0$ in your equation and solve.
To find the y-intercepts, let $x = 0$ in your equation and solve.

Example: $y = x^2 - 2x - 3$

x-int. (Let $y = 0$)

$$0 = x^2 - 2x - 3$$

$$0 = (x - 3)(x + 1)$$

$$x = -1 \text{ or } x = 3$$

x-intercepts $(-1, 0)$ and $(3, 0)$

y-int. (Let $x = 0$)

$$y = 0^2 - 2(0) - 3$$

$$y = -3$$

y-intercept $(0, -3)$

Find the x and y intercepts for each.

19. $y = 2x - 5$

20. $y = x^2 + x - 2$

21. $y = x\sqrt{16 - x^2}$

22. $y^2 = x^3 - 4x$

Use substitution or elimination method to solve the system of equations.

Example:

$$x^2 + y - 16x + 39 = 0$$

$$x^2 - y^2 - 9 = 0$$

Elimination Method

$$2x^2 - 16x + 30 = 0$$

$$x^2 - 8x + 15 = 0$$

$$(x - 3)(x - 5) = 0$$

$$x = 3 \text{ and } x = 5$$

Plug $x = 3$ and $x = 5$ into one original

$$3^2 - y^2 - 9 = 0 \quad 5^2 - y^2 - 9 = 0$$

$$-y^2 = 0 \quad 16 = y^2$$

$$y = 0 \quad y = \pm 4$$

Points of Intersection $(5, 4)$, $(5, -4)$ and $(3, 0)$

Substitution Method

Solve one equation for one variable.

$$y^2 = -x^2 + 16x - 39 \quad (\text{1st equation solved for } y)$$

$$x^2 - (-x^2 + 16x - 39) - 9 = 0 \quad \text{Plug what } y^2 \text{ is equal to into second equation.}$$

$$2x^2 - 16x + 30 = 0 \quad (\text{The rest is the same as previous example})$$

$$x^2 - 8x + 15 = 0$$

$$(x - 3)(x - 5) = 0$$

$$x = 3 \text{ or } x = 5$$

Find the point(s) of intersection of the graphs for the given equations.

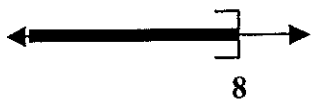
23. $x + y = 8$
 $4x - y = 7$

24. $x^2 + y = 6$
 $x + y = 4$

25. $x^2 - 4y^2 - 20x - 64y - 172 = 0$
 $16x^2 + 4y^2 - 320x + 64y + 1600 = 0$

Interval Notation

26. Complete the table with the appropriate notation or graph.

Solution	Interval Notation	Graph
$-2 < x \leq 4$		
	$[-1, 7)$	
		

Solve each equation. State your answer in BOTH interval notation and graphically.

27. $2x - 1 \geq 0$

28. $-4 \leq 2x - 3 < 4$

29. $\frac{x}{2} - \frac{x}{3} > 5$

Domain and Range

Find the domain and range of each function. Write your answer in INTERVAL notation.

30. $f(x) = x^2 - 5$

31. $f(x) = -\sqrt{x+3}$

32. $f(x) = 3 \sin x$

33. $f(x) = \frac{2}{x-1}$

Inverses

To find the inverse of a function, simply switch the x and the y and solve for the new "y" value.

Example:

$f(x) = \sqrt[3]{x+1}$	Rewrite f(x) as y
$y = \sqrt[3]{x+1}$	Switch x and y
$x = \sqrt[3]{y+1}$	Solve for your new y
$(x)^3 = (\sqrt[3]{y+1})^3$	Cube both sides
$x^3 = y+1$	Simplify
$y = x^3 - 1$	Solve for y
$f^{-1}(x) = x^3 - 1$	Rewrite in inverse notation

Find the inverse for each function.

34. $f(x) = 2x + 1$

35. $f(x) = \frac{x^2}{3}$

Also, recall that to PROVE one function is an inverse of another function, you need to show that:
 $f(g(x)) = g(f(x)) = x$

Example:

If: $f(x) = \frac{x-9}{4}$ and $g(x) = 4x+9$ show $f(x)$ and $g(x)$ are inverses of each other.

$$\begin{aligned} f(g(x)) &= 4\left(\frac{x-9}{4}\right) + 9 \\ &= x - 9 + 9 \\ &= x \end{aligned}$$

$$\begin{aligned} g(f(x)) &= \frac{(4x+9)-9}{4} \\ &= \frac{4x+9-9}{4} \\ &= \frac{4x}{4} \\ &= x \end{aligned}$$

$f(g(x)) = g(f(x)) = x$ therefore they are inverses
of each other.

Prove f and g are inverses of each other.

36. $f(x) = \frac{x^3}{2}$ $g(x) = \sqrt[3]{2x}$

37. $f(x) = 9 - x^2, x \geq 0$ $g(x) = \sqrt{9 - x}$

Equation of a line

Slope intercept form: $y = mx + b$

Vertical line: $x = c$ (slope is undefined)

Point-slope form: $y - y_1 = m(x - x_1)$

Horizontal line: $y = c$ (slope is 0)

38. Use slope-intercept form to find the equation of the line having a slope of 3 and a y-intercept of 5.
39. Determine the equation of a line passing through the point (5, -3) with an undefined slope.
40. Determine the equation of a line passing through the point (-4, 2) with a slope of 0.
41. Use point-slope form to find the equation of the line passing through the point (0, 5) with a slope of $\frac{2}{3}$.
42. Find the equation of a line passing through the point (2, 8) and parallel to the line $y = \frac{5}{6}x - 1$.
43. Find the equation of a line perpendicular to the y-axis passing through the point (4, 7).
44. Find the equation of a line passing through the points (-3, 6) and (1, 2).
45. Find the equation of a line with an x-intercept (2, 0) and a y-intercept (0, 3).

Draw and label the Unit Circle in the space below:

Continue with problem # 59.

Radian and Degree Measure

Use $\frac{180^\circ}{\pi \text{ radians}}$ to get rid of radians and convert to degrees.

Use $\frac{\pi \text{ radians}}{180^\circ}$ to get rid of degrees and convert to radians.

59. Convert to degrees: a. $\frac{5\pi}{6}$ b. $\frac{4\pi}{5}$ c. 2.63 radians

60. Convert to radians: a. 45° b. -17° c. 237°

Angles in Standard Position

61. Sketch the angle in standard position.

a. $\frac{11\pi}{6}$

b. 230°

c. $-\frac{5\pi}{3}$

d. 1.8 radians

Reference Triangles

62. Sketch the angle in standard position. Draw the reference triangle and label the sides, if possible.

a. $\frac{2}{3}\pi$

b. 225°

c. $-\frac{\pi}{4}$

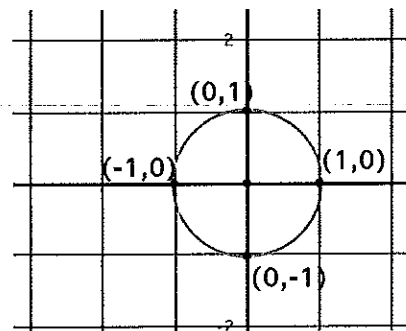
d. 30°

Unit Circle

You can determine the sine or cosine of a quadrantal angle by using the unit circle. The x-coordinate of the circle is the cosine and the y-coordinate is the sine of the angle.

Example: $\sin 90^\circ = 1$

$$\cos \frac{\pi}{2} = 0$$



63. a.) $\sin 180^\circ$

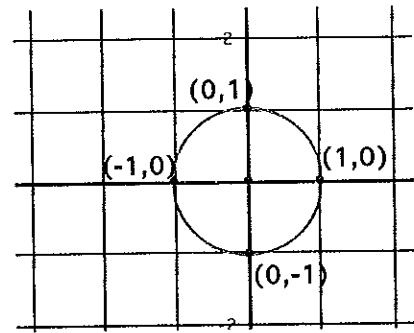
b.) $\cos 270^\circ$

c.) $\sin(-90^\circ)$

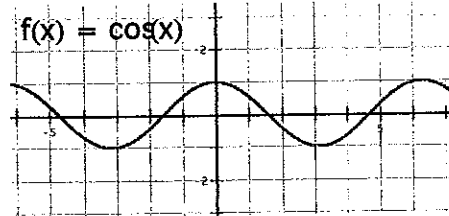
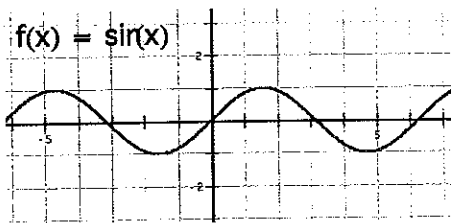
d.) $\sin \pi$

e.) $\cos 360^\circ$

f.) $\cos(-\pi)$



Graphing Trig Functions



$y = \sin x$ and $y = \cos x$ have a period of 2π and an amplitude of 1. Use the parent graphs above to help you sketch a graph of the functions below. For $f(x) = A\sin(Bx + C) + K$, A = amplitude, $\frac{2\pi}{B}$ = period, $\frac{C}{B}$ = phase shift (positive C/B shift left, negative C/B shift right) and K = vertical shift.

Graph two complete periods of the function.

64. $f(x) = 5 \sin x$

65. $f(x) = \sin 2x$

66. $f(x) = -\cos\left(x - \frac{\pi}{4}\right)$

67. $f(x) = \cos x - 3$

Trigonometric Equations:

Solve each of the equations for $0 \leq x < 2\pi$. Isolate the variable, sketch a reference triangle, find all the solutions within the given domain, $0 \leq x < 2\pi$. Remember to double the domain when solving for a double angle. Use trig identities, if needed, to rewrite the trig functions. (See formula sheet at the end of the packet.)

68. $\sin x = -\frac{1}{2}$

69. $2 \cos x = \sqrt{3}$

70. $\cos 2x = \frac{1}{\sqrt{2}}$

71. $\sin^2 x = \frac{1}{2}$

72. $\sin 2x = -\frac{\sqrt{3}}{2}$

73. $2 \cos^2 x - 1 - \cos x = 0$

74. $4 \cos^2 x - 3 = 0$

75. $\sin^2 x + \cos 2x - \cos x = 0$

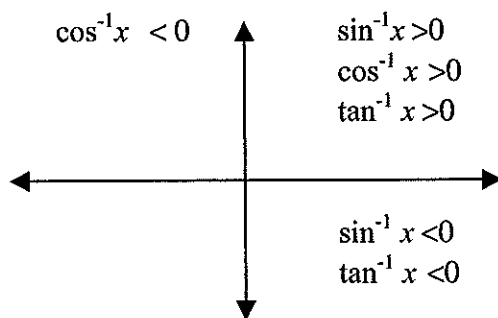
Inverse Trigonometric Functions:

Recall: Inverse Trig Functions can be written in one of ways:

$$\arcsin(x)$$

$$\sin^{-1}(x)$$

Inverse trig functions are defined only in the quadrants as indicated below due to their restricted domains.

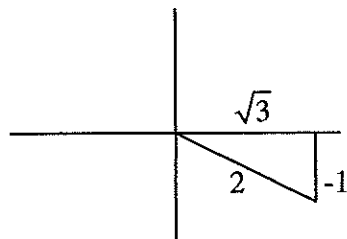


Example:

Express the value of “y” in radians.

$$y = \arctan \frac{-1}{\sqrt{3}}$$

Draw a reference triangle.



This means the reference angle is 30° or $\frac{\pi}{6}$. So, $y = -\frac{\pi}{6}$ so that it falls in the interval from

$$\frac{-\pi}{2} < y < \frac{\pi}{2}$$

$$\text{Answer: } y = -\frac{\pi}{6}$$

For each of the following, express the value for “y” in radians.

76. $y = \arcsin \frac{-\sqrt{3}}{2}$

77. $y = \arccos(-1)$

78. $y = \arctan(-1)$

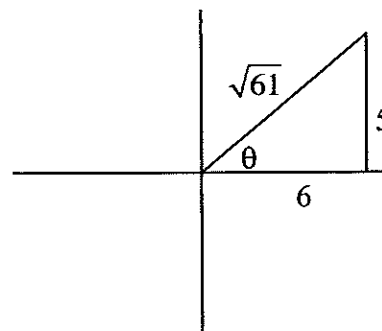
Example: Find the value without a calculator.

$$\cos\left(\arctan\frac{5}{6}\right)$$

Draw the reference triangle in the correct quadrant first.

Find the missing side using Pythagorean Thm.

Find the ratio of the cosine of the reference triangle.



$$\cos\theta = \frac{6}{\sqrt{61}}$$

For each of the following give the value without a calculator.

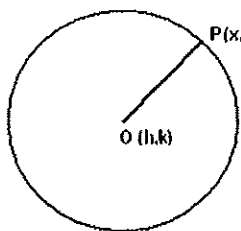
79. $\tan\left(\arccos\frac{2}{3}\right)$

80. $\sec\left(\sin^{-1}\frac{12}{13}\right)$

81. $\sin\left(\arctan\frac{12}{5}\right)$

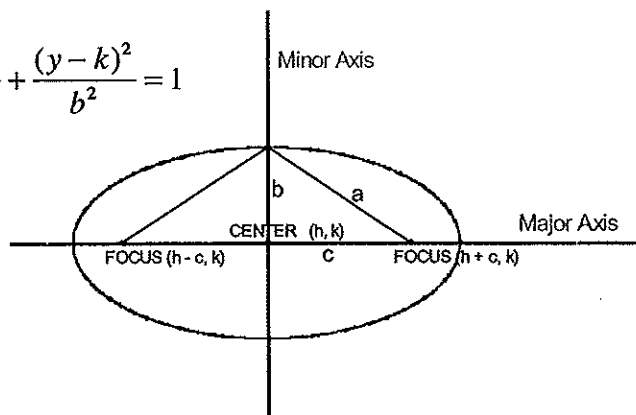
82. $\sin\left(\sin^{-1}\frac{7}{8}\right)$

Circles and Ellipses



$$r^2 = (x-h)^2 + (y-k)^2$$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

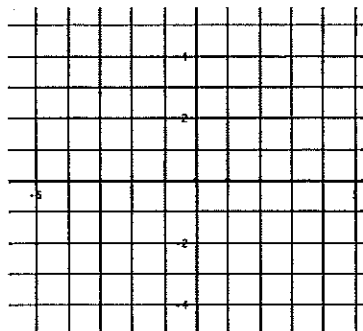


For a circle centered at the origin, the equation is $x^2 + y^2 = r^2$, where r is the radius of the circle.

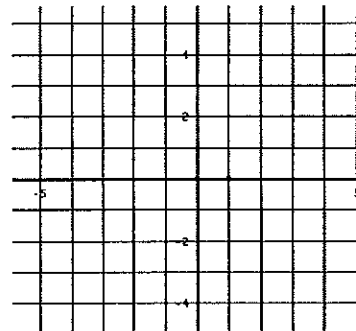
For an ellipse centered at the origin, the equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where a is the distance from the center to the ellipse along the x -axis and b is the distance from the center to the ellipse along the y -axis. If the larger number is under the y^2 term, the ellipse is elongated along the y -axis. For our purposes in Calculus, you will not need to locate the foci.

Graph the circles and ellipses below:

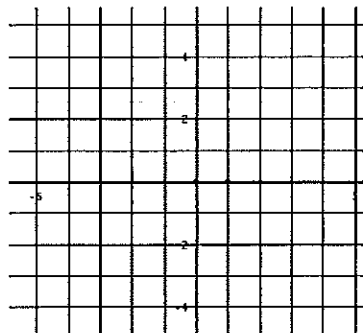
83. $x^2 + y^2 = 16$



84. $x^2 + y^2 = 5$



85. $\frac{x^2}{1} + \frac{y^2}{9} = 1$



86. $\frac{x^2}{16} + \frac{y^2}{4} = 1$

